CHAPTER 2
PREDICTION OF RADAR RANGE

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2.1 INTRODUCTION

The basic physics governing the prediction of radar maximum detection range, for a specified target under free-space conditions with detection limited by thermal noise, has been well understood since the earliest days of radar. The term *free space* implies (in the present context) that a spherical region of space, centered at the radar and extending to considerably beyond the target, is empty except for the radar and the target. (*Considerably* as used here can be precisely defined for specific radars, but a general definition would be lengthy and not very useful.) It also implies that the only radar-frequency electromagnetic waves detectable within this region, other than those emanating from the radar itself, are from natural thermal and quasi-thermal noise sources, as described in Sec. 2.5.

Although this condition is never fully realized, it is approximated for some radar situations. Under many non-free-space conditions and with radically nonthermal forms of background noise, the prediction problem is considerably more complicated. Complications not considered in early analyses also result from modification of the signal and noise relationship by the receiving-system circuitry (*signal processing*).

In this chapter the free-space equation will be presented, basic signal processing will be discussed, and some of the most important non-free-space environments will be considered. The effect of some common nonthermal types of noise will be considered. Although it will not be feasible to consider all the possible types of radar situations, the methods to be described will indicate the general nature of the necessary procedures for environments and conditions not specifically treated here. Some of the specialized types of radar, for which special analyses are required, are described in later chapters of this handbook.

Definitions. The radar range equation contains many parameters of the radar system and its environment, and some of them have definitions that are interdependent. As will be discussed in Sec. 2.3, some definitions contain an element of arbitrariness, and it is common for different authors to employ different definitions of some of the range-equation factors. Of course, when generally accepted definitions do exist, they should be observed. But even more important, although some arbitrariness may be permissible for individual definitions, once a particular definition has been adopted for one of the range-equation factors, it will be found that definition of one or more of the other factors is no longer arbitrary.

As an example, for pulsed radar the definitions of pulse power and pulse length are highly arbitrary individually, but once a definition for either one of them has been adopted, the definition of the other is determined by the constraint that their product *must* equal the *pulse energy*. In this chapter, a set of definitions that are believed to conform to such rules of consistency, as well as to definitions adopted by standards organizations, will be presented.

Conventions. Because of the wide variability of propagation-path and other range-equation factors, certain *conventions* are necessary for predicting range under standard conditions when specific values of those factors are not known. A convention is a generally accepted *standard assumption*, which may never be encountered exactly in practice but which falls within the range of conditions that will be encountered, preferably somewhere near the middle of the range. An example is the conventional geophysical assumption, for calculating certain earth environment effects that depend on the earth's curvature, that the earth is a perfect sphere of radius 6370 km. The importance of conventions is that they provide a common basis for comparison of competing radar systems. To the extent that they are fairly representative of typical conditions, they also allow prediction of a realistic detection range. Commonly accepted conventions will be used in this chapter, and where needed conventions do not exist, appropriate ones will be suggested.

Range Prediction Philosophy. It is apparent from the foregoing discussion that a range prediction based on conventional assumptions will not necessarily be confirmed exactly by experimental results. This conclusion is further warranted by the statistical nature of the "noise" which is usually the limiting factor in the signal detection process. In other words, even if all the environmental factors were precisely known, a range prediction would not be likely to be verified exactly by the result of a single experiment. A statistical prediction refers to the average result of many trials. Therefore, radar range prediction is not an exact science. (In fact, the lesson of quantum mechanics seems to be that there is no such thing as an exact science in the strict sense.)

Nevertheless, calculations to predict radar range are useful. However inexact they may be on an absolute basis, they permit meaningful comparisons of the expected relative performance of competing system designs, and they indicate the relative range performance change to be expected if the radar parameters or environmental conditions are changed. They are therefore a powerful tool for the system designer. The predicted range is a figure of merit for a proposed radar system. It is not necessarily a complete one, since other factors such as target-position-measurement accuracy, data rate, reliability, serviceability, size, weight, and cost may also be important. Despite the inexactness of range predictions in the absolute sense, the error can be made small enough that the calculated range is a good indication of the performance to be expected under average environmental conditions. Section 2.10 is a more detailed discussion of prediction accuracy.

Attempts to evaluate range prediction factors accurately, to better than perhaps 1 dB, are sometimes disparaged on the grounds that some factors are unlikely to be known with accuracy in operational situations and that hence it is useless to seek better accuracy for any factor. Although there is some basis for this viewpoint, the overall accuracy will be unnecessarily degraded if the accuracy of all the factors in the equation is deliberately reduced. Therefore it is recommended that range predictions be based on as careful an evaluation of all the factors as is possible. A goal of 0.1 dB accuracy is perhaps reasonable, although admittedly it may be impossible to evaluate all the factors in the equation with that degree of precision.

Historical Notes. Possibly the first comprehensive treatise on radar maximum-range prediction was that of Omberg and Norton,[1] published first as a U.S. Army Signal Corps report in 1943. It presents a fairly detailed range equation and contains information on evaluating some of the more problematical factors, such as multipath interference and minimum detectable signal, within the limitations of the then-available knowledge. The signal detection process was assumed to be based on visual observation of a cathode-ray-tube display. The antenna was assumed to "searchlight" the target. Statistical aspects of signal detection were not considered.

D. O. North,[2] in a classical report published with a military security classification in 1943, outlined the basic theory of a statistical treatment of signal detection. (This report was republished in *Proceedings of the IEEE*, but not until 1963.) He introduced the concepts that are now called *probability of detection* and *false-alarm probability*, and he clearly delineated the role of integration in the detection of pulse signals. This report also introduced the concept of the *matched filter*, a contribution for which it had achieved some recognition prior to 1963. But except for the matched-filter concept, its contributions to signal detection theory were virtually unrecognized by radar engineers generally until the report was republished 20 years after its first appearance.

In a famous report[3] first published in 1948 and republished in *IRE Transactions on Information Theory* in 1960, J. I. Marcum extensively developed the statistical theory of detection with the aid of machine computation, referencing North's report. He computed probabilities of detection as a function of a range parameter related to signal-to-noise ratio, for various numbers of pulses integrated and for various values of a false-alarm parameter which he designated *false-alarm number*. He employed this type of computation to study the effects of various amounts and kinds of integration, different detector (demodulator) types, losses incurred by "collapsing" one spatial coordinate on the radar display, and various other effects. His results are presented as curves for probability of detection as a function of the ratio of the actual range to that at which the signal-to-noise ratio is unity, on the assumption that the received-signal power is inversely proportional to the fourth power of the range. Since this proportionality holds only for a target in free space, application of Marcum's results is sometimes complicated by this mode of presentation.

Marcum considered only *steady* signals (target cross section not varying during the period of observation), and most of his results assume the use of a square-law detector. Robertson[4] has published exceptionally detailed and useful steady-signal results applicable to the linear-rectifier detector, which is the type of detector almost universally used. (The square-law-detector results are also useful because they differ very little from the linear-detector results.) Swerling extended Marcum's work to include the case of fluctuating signals.[5] His report was republished in *IRE Transactions on Information Theory* in 1960. Fehlner[6] recomputed Marcum's and Swerling's results and presented them in the more useful form of curves with the signal-to-noise power ratio as the abscissa. The fluctuating-signal problem has subsequently been further treated by Kaplan,[7] Schwartz,[8] Heidbreder and Mitchell,[9] Bates,[10] and others.

Hall[11] published in 1956 a comprehensive paper on radar range prediction in which the concepts of probability of detection, false-alarm probability, the relative effects of predetection and postdetection integration, and the effects of scanning the antenna beam were considered. The range equation was formulated in terms of an ideal (matched-filter) utilization of the available received-signal power, with loss factors to account for departures from the ideal.

Blake[12] published an updating of the subject in 1961, applying recent advances in system-noise-temperature calculation, atmospheric absorption, plotting of coverage diagrams based on a realistic atmospheric refractive-index model, and multipath-interference calculation. This work was followed by Naval Research Laboratory (NRL) reports[13] and a book[14] in which further details were presented.

Contributions to the subject of range prediction have also been made by many others, far too numerous to mention by name. Only the major contributions can be recognized in this brief history. Special mention should be made, however, of the many contributions in two volumes (13 and 24) of the MIT Radiation Laboratory Series, edited by Kerr[15] and by Lawson and Uhlenbeck.[16] Much use is made in this chapter of results originally published in those volumes.

2.2 RANGE EQUATIONS

Radar Transmission Equation. The following equation, in the form given in Kerr,[15] is called the *transmission equation* for monostatic radar (one in which the transmitter and receiver are colocated):

$$\frac{P_r}{P_t} = \frac{G_t G_r \sigma \lambda^2 F_t^2 F_r^2}{(4\pi)^3 R^4} \tag{2.1}$$

where P_r = received-signal power (at antenna terminals)
P_t = transmitted-signal power (at antenna terminals)
G_t = transmitting-antenna power gain
G_r = receiving-antenna power gain
σ = radar target cross section
λ = wavelength
F_t = pattern propagation factor for transmitting-antenna-to-target path
F_r = pattern propagation factor for target-to-receiving-antenna path
R = radar-to-target distance (range)

This equation is not identical to Kerr's; he assumes that the same antenna is used for transmission and reception, so that $G_t G_r$ becomes G^2 and $F_t^2 F_r^2$ becomes F^4. The only factors in the equation that require explanation are the pattern propagation factors F_t and F_r. The factor F_t is defined as the ratio, at the target position, of the field strength E to that which would exist at the same distance from the radar in free space and in the antenna beam maximum-gain direction, E_0. The factor F_r is analogously defined. These factors account for the possibility that the target is not in the beam maxima (G_t and G_r are the gains in the maxima) and for any propagation gain or loss that would not occur in free space. The most common of these effects are absorption, diffraction and shadowing, certain types of refraction effects, and multipath interference.

For a target in free space and in the maxima of both the transmit and receive antenna patterns, $F_t = F_r = 1$. Detailed definitions of these and other range-equation factors are given in Secs. 2.3 to 2.7.

Maximum-Range Equation. Equation (2.1) is not a range equation as it stands, although it can be rewritten in the form

$$R = \left[\frac{P_t G_t G_r \sigma \lambda^2 F_t^2 F_r^2}{(4\pi)^3 P_r}\right]^{1/4} \tag{2.2}$$

This equation states that R is the range at which the received-echo power will be P_r if the transmitted power is P_t, target size σ, and so forth. It becomes a maximum-range equation by the simple expedient of attaching subscripts to P_r and R so that they become $P_{r,\min}$ and $R_{\max}$. That is, when the value of P_r in Eq. (2.2) is the minimum detectable value, the corresponding range is the maximum range of the radar.

However, this is a very rudimentary maximum-range equation, of limited usefulness. A first step toward a more useful equation is replacement of P_r by a more readily evaluated expression. This is done by first defining the signal-to-noise power ratio:

$$S/N = P_r/P_n \tag{2.3}$$

where P_n is the power level of the noise in the receiving system, which determines the minimum value of P_r that can be detected. This noise power, in turn, can be expressed in terms of a receiving-system noise temperature T_s:

$$P_n = kT_sB_n \tag{2.4}$$

where k is Boltzmann's constant (1.38×10^{-23} Ws/K) and B_n is the noise bandwidth of the receiver predetection filter, hertz. (These quantities are defined more completely in Secs. 2.3 and 2.5.[17]) Therefore,

$$P_r = (S/N)\, kT_sB_n \tag{2.5}$$

This expression can now be substituted for P_r in Eq. (2.2).

A further convenient modification is to redefine P_t as the transmitter power at the terminals of the transmitter, rather than [as in Eq. (2.1)] the usually somewhat smaller power that is actually delivered to the antenna terminals because of loss in the transmission line. This redefinition is desirable because when radar system designers or manufacturers specify a transmitter power, the actual transmitter output power is usually meant.

With this changed definition, P_t must be replaced by P_t/L_t, where L_t is a *loss factor* defined as the ratio of the transmitter output power to the power actually delivered to the antenna. (Therefore, $L_t \geqq 1$.)

It will later prove convenient to introduce additional loss factors similarly related to other factors in the range equation. These loss factors are multiplicative;

that is, if there are, for example, three loss factors L_1, L_2, and L_3, they can be represented by a single *system loss factor* $L = L_1L_2L_3$. The resulting maximum-range equation is

$$R_{\max} = \left[\frac{P_tG_tG_r\sigma\lambda^2F_t^2F_r^2}{(4\pi)^3(S/N)_{\min}kT_sB_nL}\right]^{1/4} \tag{2.6}$$

The quantities $(S/N)_{\min}$ and T_s as here defined are to be evaluated at the antenna terminals, and that fact detracts from the utility of this form of the equation. As thus defined, $(S/N)_{\min}$ is not independent of B_n, and the dependence is difficult to take into account in this formulation. If that dependence were ignored, this equation would imply that $R_{\max}$ is an inverse function of B_n; i.e., if all the other range-equation factors were held constant, $R_{\max}$ could be made as large as desired simply by making B_n sufficiently small. This is well known to be untrue. To remedy this difficulty, several factors must be considered. It is convenient to do this in terms of a particular transmitted waveform.

Pulse Radar Equation. Equation (2.6) does not specify the nature of the transmitted signal; it can be CW (continuous-wave), amplitude- or frequency-modulated, or pulsed. It is advantageous to modify this equation for the specific case of pulse radars and in so doing to remove the "bandwidth" difficulty encountered in using Eq. (2.6). Pulse radars are of course the most common type. As will be shown, although the equation thus modified will ostensibly be restricted to pulse radars, it can in fact be applied to other types of radar by appropriate reinterpretation of certain parameters.

D. O. North[2] demonstrated that the detectable signal-to-noise ratio $(S/N)_{\min}$ will have its smallest possible value when the receiver bandwidth B_n has a particular (optimum) value and that this optimum value of B_n is inversely proportional to the pulse length τ. This implies that an equation can be written with pulse length in the numerator rather than with bandwidth in the denominator. North also showed that signal detectability is improved by *integrating* successive signal and noise samples in the receiver and that the detectability is a function of the total integrated signal energy. (The integration process is discussed in Sec. 2.4.) Finally, he showed that *when the receiver filter is matched to the pulse waveform*, the ratio of the received-pulse energy to the noise power spectral density at the output of the receiver filter is maximized *and is equal to the signal-to-noise power ratio at the antenna terminals*. The term *matched* in this context means, partially, that the filter bandwidth is optimum. The full meaning is that the filter transfer function is equal to the complex conjugate of the pulse spectrum.

Detectability Factor. An equation based on these facts can be derived by utilizing a parameter called *detectability factor*, defined by the Institute of Electrical and Electronics Engineers (IEEE)[18] as follows: "In pulsed radar, the ratio of single-pulse signal energy to noise power per unit bandwidth that provides stated probabilities of detection and false alarm, measured in the intermediate-frequency amplifier and using an intermediate-frequency filter matched to the single pulse, followed by optimum video integration." Deferring for the moment discussion of the meaning of some aspects of this definition, it can be expressed mathematically as follows:*

$$D_0 = E_r/N_0 = P_t\tau/kT_s \tag{2.7}$$

where D_0 is the detectability factor, E_r is the received-pulse energy, and N_0 is the noise power per unit bandwidth, both measured at the output of the receiver filter (i.e., at the demodulator input terminals).

The next step in this reformulation of the range equation is to define a bandwidth correction factor C_B, to allow for the possibility that the receiver filter bandwidth B_n may not be optimum. This factor is defined by the following relationship:

$$(S/N)_{\min}B_n = (S/N)_{\min(0)}B_{n,\,\text{opt}}C_B = D_0B_{n,\,\text{opt}}C_B \tag{2.8}$$

where $B_{n,\text{opt}}$ is the optimum value of B_n. The factor C_B has been named the *bandwidth correction factor* because it was originally defined in terms of bandwidth optimization, but in actuality it is a *filter mismatch factor*, in the North matched-filter sense. As Eq. (2.8) implies, $C_B \geqq 1$. Evaluation of C_B is discussed in Sec. 2.3.

The quantity $(S/N)_{\min(0)}$ in Eq. (2.8) is the optimum-bandwidth (matched-filter) value of $(S/N)_{\min}$, which North showed to be equal to D_0. It is this fact that allows the range equation to be written, as desired, in terms of the signal-to-noise ratio at the detector input terminals (filter output) rather than the ratio at the antenna terminals.

North deduced that $B_{n,\text{opt}} = 1/\tau$ exactly. As will be discussed later, some radar detection experiments with human observers have subsequently suggested that the constant of proportionality may not be exactly unity. However, North's analysis is theoretically correct for pulses of rectangular shape and for the definition to be given in Sec. 2.3 for the noise bandwidth B_n. For pulses of other shapes the pulse-length–bandwidth relationship is subject to the particular definition used for the pulse length. That definition is not an issue, of course, when the pulse shape is rectangular.

Based on that result, the range equation can be written with pulse length in the numerator by means of the following equivalence, in terms of the parameters of Eq. (2.8):

$$(S/N)_{\min}B_n = D_0C_B/\tau \tag{2.9}$$

The expression of the left-hand side of Eq. (2.9), where it occurs in the denominator of Eq. (2.6), can now be replaced by the expression of the right-hand side. The result is the desired pulse radar form of the range equation:

$$R_{\max} = \left[\frac{P_t\tau G_tG_r\sigma\lambda^2F_t^2F_r^2}{(4\pi)^3kT_sD_0C_BL}\right]^{1/4} \tag{2.10}$$

*In some of the literature it is stated that the matched-filter output signal-to-noise ratio is $2E_r/N_0$. That statement is based on defining peak signal power as the instantaneous value occurring not only at the peak of the output-pulse waveform but also at the peak of an RF cycle, where the instantaneous power is theoretically twice the average power. North's definition, based on the signal power averaged over an RF cycle, is consistent with the definition of noise power as the average over both the RF cycles and the random noise fluctuations.

A primary advantage of this formulation of the equation is that standard curves for the parameter D_0, as a function of the number of pulses integrated, are available, with the probabilities of detection and false alarm as parameters (Sec. 2.4). Calculation of these curves is necessarily done in terms of D_0, the signal-to-noise ratio at the demodulator input terminals.

The emphasis of this equation on the significance of the pulse *energy* (the product $P_t\tau$ in the numerator) is valuable to the system designer. It also provides a simple answer to the question of which pulse length to use in the range equation when the radar employs *pulse compression*, in which a coded pulse waveform of relatively long duration is transmitted and then "compressed" to a short pulse upon reception. The correct answer is deduced from the fact that the product $P_t\tau$ must equal the transmitted pulse energy. Therefore if the pulse power P_t is the power of the long (uncompressed) transmitted pulse, then τ must be the duration of that pulse.

A further advantage of this form of the equation, or more specifically of the definition of the detectability factor, is the indicated dependence of the radar detection range on the *integration* of successive pulses, if any, that takes place in the receiving system. Integration is discussed in Sec. 2.4.

Finally, as was mentioned earlier, this formulation of the range equation, although derived specifically in terms of pulse radar parameters, can be applied to CW radars and to radars that utilize forms of signal modulation other than pulses. Its application to these other radar types is accomplished by redefining the parameters τ and D_0. Details of this procedure are presented in Ref. 14, Chaps. 2 and 9.

Probabilistic Notation. It has been mentioned (Sec. 2.1) that the radar signal detection process is basically probabilistic or statistical in nature. This results from the nature of the noise voltage that is always present in the receiver circuits. This voltage is randomly varying or fluctuating, and when it is intermixed with a radar echo signal, it becomes impossible to tell with certainty whether a momentary increase of the receiver output is due to a signal or to a chance noise fluctuation. However, it is possible to define probabilities for these two possibilities and to discuss the detection process in terms of them in a quantitative manner. The probability that the signal, when present, will be detected is called the *probability of detection,* P_d, and the probability that a noise fluctuation will be mistaken for a signal is called the *false-alarm probability,* P_{fa}.

The notations R_{max}, $P_{r,min}$, and $(S/N)_{min}$ can then be replaced by more precise notation, using subscripts to denote the applicable values of P_d and P_{fa}. However, the *fa* subscript is ordinarily suppressed, though implied. Thus R_{50} can denote the range for 0.5 (i.e., 50 percent) probability of detection and some separately specified false-alarm probability.

If the target cross section σ fluctuates, this fluctuation will alter the signal-plus-noise statistics. As mentioned in Sec. 2.1, this problem has been analyzed by Swerling[5] and others.[6–10] Curves have been calculated that allow determining the appropriate value of D_0 for the fluctuating-signal case, for various probabilities of detection and false alarm (Sec. 2.4).

Automatic Detection. Detection* is said to be *automatic* if the decision concerning the presence or absence of a received signal is made by a purely

*A note on various meanings of the words *detect, detector,* and *detection* is desirable here. In radio usage, a detector has come to mean either a frequency converter (e.g., a superheterodyne first detector) or a demodulator (often the "second detector" of a superheterodyne receiver, which is usually a linear rectifier). Then, *detection* means the waveform modification produced by such a device. An *automatic* physical device, without direct human intervention. Such a device, described by North,[2] establishes a threshold voltage level (for example, by means of a biased diode). If the processed (e.g., integrated) receiver output exceeds the threshold (as evidenced by diode current flow), some mechanism is actuated to indicate this fact in an unequivocal fashion. The mechanism could be the lighting of a light, the ringing of a bell, or more generally the setting of a bit to 1 in a binary data channel wherein a 0 corresponds to no signal. Various additional consequences may then automatically ensue. The analysis of radar detection can thus be regarded as a problem in statistical decision theory.

Bistatic Radar Equation. The foregoing equations assume that the transmitting and receiving antennas are at the same location (monostatic radar). A bistatic radar (Chap. 25) is one for which the two antennas are widely separated, so that the distance and/or the direction from the transmitting antenna to the target are not necessarily the same as the distance and/or direction from the receiving antenna to the target. Moreover, since the signal reflected from the target to the receiving antenna is not directly backscattered, as it is for monostatic radar, the target cross section is not usually the same (for a given target viewed in a given aspect by the transmitting antenna). A *bistatic radar cross section* σ_b is defined to apply for this situation. The symbol σ in the preceding equations implies a monostatic cross section. Range equations for bistatic radar are obtained from the foregoing monostatic equations by replacing the range R and the target cross section σ by the corresponding bistatic quantities. The bistatic equivalent of R is $\sqrt{R_t R_r}$, where R_t is the distance from the transmitting antenna to the target and R_r is the distance from the target to the receiving antenna.

Equations in Practical Units. The equations that have been given are valid when a consistent system of units is used, such as the rationalized meter-kilogram-second (mks) system. In many applications, however, it is convenient or necessary to employ "mixed" units. Moreover, it is usually more convenient to express the wavelength λ in terms of the equivalent frequency in megahertz. It is also desirable to combine all the numerical factors and the various unit-conversion factors into a single numerical constant. For a particular system of mixed units, the following equation is obtained from Eq. (2.10):

$$R_{\max} = 129.2\left[\frac{P_{t(\mathrm{kW})}\tau_{\mu\mathrm{s}}G_tG_r\sigma F_t^2F_r^2}{f_{\mathrm{MHz}}^2T_sD_0C_BL}\right]^{1/4} \tag{2.11}$$

The subscript notation $R_{\max}$ is now meant to imply the range corresponding to specified detection and false-alarm probabilities. For this equation, the range is given in international nautical miles. (One international nautical mile is exactly 1852 m.) The target cross section σ is in square meters, transmitter power P_t in kilowatts, pulse length τ in microseconds, frequency f in megahertz, and system noise temperature T_s in kelvins. (All other quantities are dimensionless.)

If the range is desired in units other than nautical miles (all other units remaining the same), in place of the factor 129.2 the following numerical constants should be used in Eq. (2.11):

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detector, however, is a decision-making device—for example, a device that replaces the human observer of a cathode-ray-tube display—and in that context *detection* is the making of a positive decision. In this chapter the meaning will ordinarily be evident from the context. Where confusion might otherwise result, the term *detection-decision device* may be used to denote an automatic detector.

Range units	Constant, Eq. (2.11)
Statute miles	148.7
Kilometers	239.3
Thousands of yards	261.7
Thousands of feet	785.0

A decibel-logarithmic form of the range equation is sometimes useful. An equation of that type, corresponding to Eq. (2.11), is readily obtained as the algebraic sum of the logarithms of the terms of that equation (with appropriate multipliers for the decibel format and for exponents), since it involves only multiplication, division, and exponentiation.

2.3 DEFINITION AND EVALUATION OF RANGE FACTORS

There is an element of arbitrariness in the definition of most of the factors of the radar range equation, and for some of them more than one definition is in common use. Since the definitions in these cases are arbitrary, one definition is in principle as good as another. However, once a definition has been chosen for one factor, there is no longer freedom of choice for one or more of the others. The factors are interdependent, and mutual compatibility is essential. A set of definitions that are believed to be mutually compatible will be given here. Also, information needed for evaluating these factors will be given insofar as is practicable. Certain range-equation factors that present special problems will be considered at greater length in subsequent sections of the chapter.

Transmitter Power and Pulse Length. The radar transmission equation, from which all the subsequent range equations are derived, is an equation for the dimensionless ratio P_t/P_r. Consequently, the most basic requirement on the definition of P_t is that it agree with the definition of P_r. For a CW radar, the power (averaged over an RF cycle) is constant, and there is no definition problem. For a pulse radar, both P_t and P_r are usually defined as the *pulse power*, which is the *average power during the pulse*. More precisely,

$$P_t = \frac{1}{\tau} \int_{T/2}^{-T/2} W(t)\, dt \qquad (2.12)$$

where $W(t)$ is the instantaneous power (a function of time, t). The definition of $W(t)$, however, excludes "spikes," "tails," and any other transients that are not useful for radar detection. The time interval T is the pulse period (=1/PRF, where PRF is the pulse repetition frequency in pulses per second). Because of the exclusion of nonuseful portions of the waveform (as it exists at the transmitter output terminals), P_t as thus defined may be called the *effective* pulse power. It is often referred to as the *peak power*. However, peak power more properly signifies the power level at the peak of the pulse waveform (averaged over an RF cycle), and *pulse power* is more appropriate.

In the transmission equation, Eq. (2.1), P_t and P_r are the transmitted and received powers at the antenna terminals. As was mentioned in Sec. 2.2, P_t is now defined at the *transmitter output* terminals, and any loss between these terminals and the antenna input terminals must be expressed as a loss factor L_t.

The pulse power P_t and the pulse length τ must be defined so that their product is the pulse energy. Any definition of τ will produce this result if the same definition is used in Eq. (2.12). The customary definition, and the one recommended here, is the time duration between the half-power points of the envelope of the RF pulse (0.707-V points). For some purposes, such as analyzing the range resolution or accuracy, this arbitrary definition of the pulse length is not permissible. But the half-power definition is customary and acceptable for use in the range equation.

The range equation can be written with the product $P_t\tau$ replaced by the pulse energy E_t. The more detailed notation is used here because, for ordinary pulse radars, P_t and τ are usually given explicitly and E_t is not. However, the use of E_t in the equation does have the advantage of avoiding the problems of defining P_t and τ; and it is especially useful when complicated waveforms are transmitted.

If *coherent integration for a fixed integration time* is assumed, the equation can also be written with the transmitted *average* power in the numerator. For simple pulse radars, the average power is the product of pulse power, pulse length, and pulse repetition frequency. In this average-power formulation, the average power $\bar{P}_t$ is multiplied by the integration time t_i (assumed to be long compared with the interpulse period) to obtain the transmitted energy. Then the value of D_0 used is that which would apply if detection were based on observation of a single pulse. (See Sec. 2.4, Fig. 2.3.) The average-power formulation is especially useful for CW or pulse doppler radars.

Antenna Gain, Efficiency, and Loss Factor. The gains G_t and G_r are defined as the power gains of the antennas *in the maximum-gain direction*. If a target of interest is at an elevation angle not in the beam maxima, that fact is accounted for by the pattern propagation factors F_t and F_r, discussed in Sec. 2.6. The maximum power gain of an antenna is equal to its directivity (maximum directive gain) multiplied by its radiation efficiency.[19] The directivity D is defined in terms of the electric-field-strength pattern $E(\theta,\phi)$ by the expression

$$D = \frac{4\pi E^2_{\max}}{\int_0^{2\pi}\int_0^{\pi} E^2(\theta,\phi)\sin\theta\, d\theta\, d\phi} \tag{2.13}$$

where θ and ϕ are the angles of a spherical-coordinate system whose origin is at the antenna and $E_{\max}$ is the value of $E(\theta,\phi)$ in the maximum-gain direction.

The radiation efficiency of the transmitting antenna is the ratio of the power input at the antenna terminals to the power actually radiated (including minor-lobe radiation). In terms of the receiving antenna, the equivalent quantity is the ratio of the total signal power extracted from the incident field by the antenna, with a matched-load impedance, to the signal power actually delivered to a matched load. The reciprocal of the radiation efficiency is the antenna loss factor L_a, which plays a part in the calculation of antenna noise temperature (Sec. 2.5).

Measured antenna gains are usually power gains, whereas gains calculated from pattern measurements or theory are directive gains. If the antenna gain figures supplied for use in the range equations of this chapter are of the latter type, they must be converted to power gains by dividing them by the appropriate loss factor. For many simple antennas the ohmic losses are negligible, and in those cases the power gain and the directive gain are virtually equal. However, this is by no means a safe assumption in the absence of specific knowledge. Array antennas in particular are likely to have significant ohmic losses in waveguides or coaxial lines used to distribute the power among the radiating elements.

If separate transmitting and receiving antennas are used and if their maximum gains occur at different elevation angles (this is a possible though not a common situation), appropriate correction can be made by means of the pattern factors $f_t(\theta)$ and $f_r(\theta)$, contained in the pattern propagation factors F_t and F_r (Sec. 2.6).

Antenna Beamwidth. This property of the antenna does not appear explicitly in the range equations, but it affects the range calculation through its effect on the number of pulses integrated when the antenna scans. The conventional definition is the angular width of the beam between the half-power points of the pattern. *Pattern* is used here in the usual antenna sense, for one-way transmission. It is not the two-way pattern of the radar echo signal from a stationary target as the antenna scans past it.

If a radar target, as viewed from the radar antenna, has an angular dimension that is appreciable compared with the beamwidth, the target cross section becomes a function of the beamwidth (see Sec. 2.8). For computing an effective value of σ in this case, in principle a special definition of beamwidth is needed (Ref. 15, p. 483). For practical work, however, the error that results from using the half-power beamwidth in this application is usually acceptable.

Target Cross Section. The definition of *radar target cross section* that applies for use in the foregoing radar range equations is given in Chap. 11, and the reader is referred to that chapter for a detailed discussion of the subject. Here mention will be made of a few aspects of the definitions that are of particular significance to the range prediction problem.

Targets can be classified as either *point targets* or *distributed targets*. A point target is one for which (1) the maximum transverse separation of significant scattering elements is small compared with the length of the arc intercepted by the antenna beam at the target range and (2) the maximum radial separation of scattering elements is small compared with the range extent of the pulse. At distance R from the antenna, the transverse dimension of the antenna beam is R times the angular beamwidth in radians. The range extent of the pulse is $c\tau/2$, where c is the speed of wave propagation in free space, 3×10^5 km/s, and τ is the pulse duration in seconds. Most of the targets for which range prediction is ordinarily of interest are point targets, e.g., aircraft at appreciable distances from the radar.

However, range predictions for distributed targets are sometimes wanted. The moon, for example, is a distributed target if the radar beamwidth is comparable to or less than 0.5° or if the pulse length is less than about 11.6 ms. A rainstorm is another example of a distributed target. Often, distributed targets are of interest because echoes from them (called *radar clutter*) tend to mask the echoes from the point targets whose detection is desired (see Sec. 2.8). Echoes from rain may be regarded as clutter when they interfere with detection of aircraft or other point targets, but they are themselves the signals of prime interest for weather radar.

The radar range equation is derived initially for a point target, and when that equation or the subsequent equations derived from it are used to predict the detection range for distributed targets, complications arise. In many cases, however, the point-target equation can be used for distributed targets by employing a suitable "effective" value of σ (Sec. 2.8).

The cross section of any nonspherical target is a function of the aspect angle from which it is viewed by the radar. It may also be a function of the polarization of the radar electromagnetic field. Therefore, in order to be wholly meaningful, a radar range prediction for a specific target, such as an aircraft, must stipulate the target aspect angle assumed and the polarization employed. Ordinarily, the nose aspect of an aircraft (approaching target) is of principal interest. The commonly used polarizations are horizontal, vertical, and circular. Tabulations of radar cross-section measurements of aircraft sometimes give nose, tail, and broadside values.

If the values are obtained from *dynamic* (moving-target) measurements, they are usually time averages of fluctuating values; otherwise they are *static* values for a particular aspect. Since the *instantaneous* cross section of a target is a function of the aspect angle, targets that are in motion involving random changes of aspect have cross sections that fluctuate randomly with time, as was mentioned in Sec. 2.2. This fluctuation must be taken into account in the calculation of probability of detection, as will be discussed in Sec. 2.4. When σ fluctuates, the value to be used in the range equation as formulated here is the time average.

Because of the wide variation of cross-section values of real targets, the range performance of a radar system is often stated for a particular target–cross-section assumption. A favorite value for many applications is 1 m^2. This represents the approximate cross section of a small aircraft, nose aspect, although the range for different "small" aircraft may be from less than 0.1 m^2 to more than 10 m^2. Radars are often performance-tested by using a metallic sphere, sometimes carried aloft by a free balloon, as the target because the cross section of a sphere can be accurately calculated and it does not vary with the aspect angle or the polarization.

A special definition problem arises when the target is large enough to be nonuniformly illuminated by the radar. A ship, for example, may be tall enough so that the pattern propagation factor F has different values from the waterline to the top of the mast. This matter is discussed in Ref. 15, p. 472 ff.

Wavelength (Frequency). There is ordinarily no problem in definition or evaluation (measurement) of the frequency to be used in the radar range equation. However, some radars may use a very large transmission bandwidth, or they may change frequency on a pulse-to-pulse basis, so that a question can exist as to the frequency value to be used for predicting range. Also, the presence of f (or λ) in the range equations makes it clear that the range can be frequency-dependent, but the exact nature of the frequency dependence is not always obvious because other factors in the range equation are sometimes implicitly frequency-dependent. Therefore an analysis of how the range depends on frequency can be rather complicated, and the answer depends partly on what factors are regarded as frequency-dependent and which ones are held constant as the frequency is changed. For example, most antennas have gain that is strongly frequency-dependent, but some antenna types are virtually frequency-independent over a fairly wide frequency band.

Bandwidth and Matching Factors. The frequency-response width (bandwidth) of the receiver selective circuits appears explicitly in Eqs. (2.4) to (2.6), but it is an implicit factor in the other range equations as well, through the factor C_B. From Eq. (2.4) it is clear that B_n directly affects the noise level in the receiver output. In general it also affects the signal, but not necessarily in the same manner as the noise is affected, because the signal spectrum is not usually uniform. There is a value of B_n that optimizes the output signal-to-noise ratio, as indicated by Eq. (2.8), and this optimum bandwidth is inversely proportional to the pulse length τ. (This statement applies to pulse compression radars as well as to others if τ is, *in this context*, the *compressed* pulse length, since it is the compressed pulse that is amplified in the receiver. However, as has been emphasized in Sec. 2.2, in the numerator of the range equation the *uncompressed* pulse length must be used along with the actual radiated pulse power, P_t.)

Since the range equation (2.6) and those subsequently derived from it incorporate the assumption of Eq. (2.4) (namely, that the noise output power of the receiver is equal to $kT_sB_nG_0$), the definition of B_n—the *noise bandwidth*—must conform to that assumption. The resulting correct definition, due to North,[20] is

$$B_n = \frac{1}{G_0}\int_0^\infty G(f)df \tag{2.14}$$

where G_0 is the gain at the nominal radar frequency and $G(f)$ is the frequency–power gain characteristic of the receiver predetection circuits, from antenna to detector.

The definition specifies $G(f)$ to be the response characteristic of the predetection circuits only. That is because for maximum postdetection signal-to-noise ratio the video bandwidth should be equal to at least half of the predetection bandwidth; and if it is of this width or wider, its exact width has little or no effect on signal detectability (Ref. 16, p. 211 ff.).

It is common practice to describe receiver bandwidth as the value between half-power points of the frequency-response curve. Fortunately, this value is usually very close to the true noise bandwidth, although the exact relationship of the two bandwidths depends on the particular shape of the frequency-response curve (Ref. 16, p. 177).

The bandwidth correction factor C_B in Eqs. (2.10) and (2.11) accounts for the fact that if B_n is not the optimum value, a value of signal-to-noise ratio larger than the optimum-bandwidth value D_0 is required. Therefore $C_B \geqq 1$. From data obtained in signal detection experiments during World War II at the Naval Research Laboratory, Haeff[21] devised the following empirical expression:

$$C_B = \frac{B_n\tau}{4\alpha}\left[1 + \frac{\alpha}{B_n\tau}\right]^2 \tag{2.15}$$

where B_n is the noise bandwidth, τ is the pulse length, and α is the product of τ and $B_{n,\mathrm{opt}}$ (optimum bandwidth). Figure 2.1 is a plot of Haeff's equation.

Actually, Haeff deduced from his experiments, as did North from theoretical analysis, that $B_{n,\mathrm{opt}} = 1/\tau$; that is, $\alpha = 1$, for rectangular-shaped pulses. How-

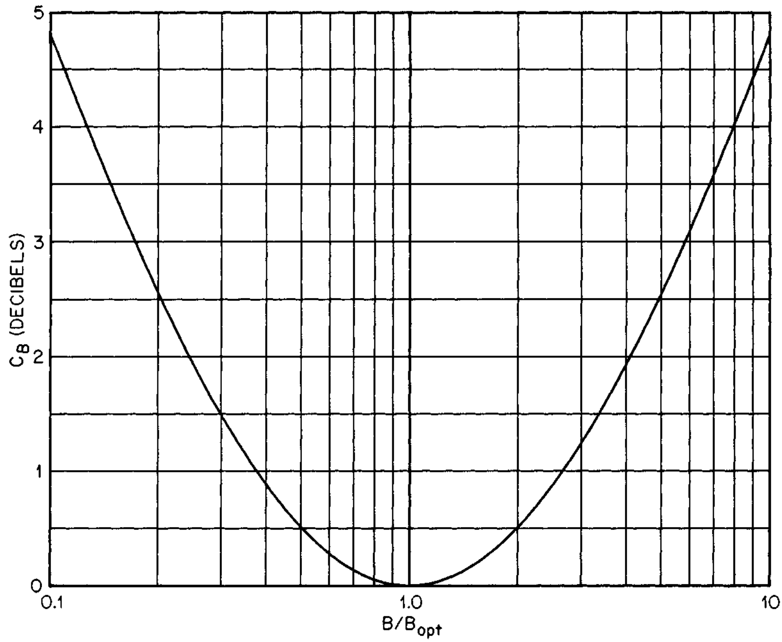

FIG. 2.1 Bandwidth correction factor C_B as a function of bandwidth B_n relative to optimum bandwidth B_{opt}; plotted from Haeff's empirical formula, Eq. (2.15).

ever, on the basis of experiments at the Massachusetts Institute of Technology (MIT) Radiation Laboratory conducted somewhat later (also with rectangular pulses), it was concluded (Ref. 16, p. 202) that $\alpha = 1.2$ for detection of signals by visual observation of cathode-ray-tube displays. Figure 2.2 is a plot of the Radiation Laboratory experimental results. The value 1.2 has subsequently been widely used for determining $B_{n,opt}$ in radar design and for computing C_B in radar range prediction.[11,12] However, North* has suggested that the $\alpha = 1.2$ figure may be based on a misinterpretation of the Radiation Laboratory data. Also, it has been noted† that the number of actual data points in Fig. 2.2 may be too few on which to base a good estimate of the optimum. Consequently, it is possible that the value of α for human observation of visual displays is much closer to 1 than was deduced by Lawson and Uhlenbeck. Fortunately, the minima of the curves (Fig. 2.2) are very broad and flat, and therefore the exact value of α does not make much difference for the usual range of values of $B_n\tau$.

The interpretation of C_B as a factor that accounts only for nonoptimum *width* of the predetection filter is permissible for simple pulse shapes and approximate results, but in principle it must also account for the complete amplitude-phase characteristic of the filter: its departure from a matched-filter characteristic. The

*In a private communication to the author in 1963.

†By M. I. Skolnik, editor of this handbook, in his review of this chapter.

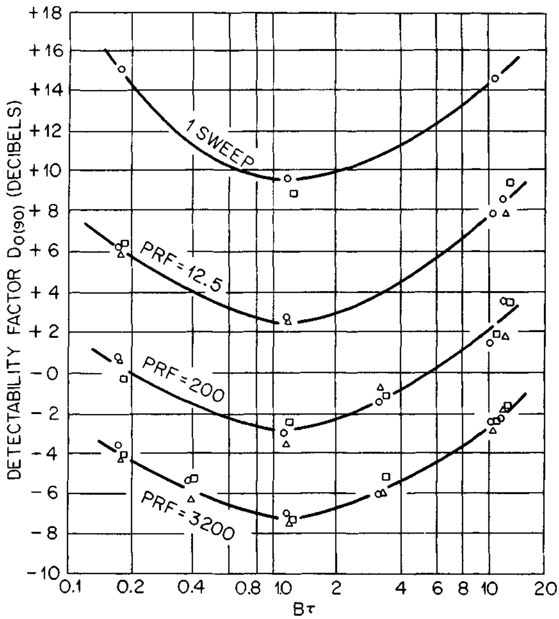

FIG. 2.2 Experimental results showing the effect of bandwidth (parameter $B\tau$) on 90 percent probability detectability factor $D_{0(90)}$, with pulse repetition frequency (PRF) as a parameter. The experiments were performed during World War II at the MIT Radiation Laboratory. *(From Ref. 15, Fig. 8.7.)*

matched-filter condition as stated by North[2] is that the receiver transfer characteristic must be the complex conjugate of the spectrum of the echo at the receiving-antenna terminals.

2.4 MINIMUM DETECTABLE SIGNAL-TO-NOISE RATIO

In Sec. 2.3, factors in the range equations were defined, and some information on how to evaluate them in typical cases was given. However, several very important factors were not covered because they are of sufficient importance to warrant more extensive treatment in separate sections. In this section and in Secs. 2.5 to 2.7, these additional factors will be discussed.

The quantities $P_{r,\min}$, $(S/N)_{\min}$, and D_0 are all related, as indicated in the development of Eqs. (2.3) to (2.9). Determination of the appropriate numbers to use for these quantities in their respective equations is a basic problem of radar range prediction. As will be seen, one of the problems is to define the meaning of *detectable*.

Integration of Signals. Detection of radar echo signals is usually (with some exceptions) accomplished by first integrating (e.g., adding) a sequence of received pulses and basing the detection decision on the resultant integrated signal voltage. Integrators that perform this operation will of course necessarily add noise as well as signals, but it is demonstrable that the ratio of the added signal voltages to the added noise voltages will be greater than the preintegration signal-to-noise ratio. Stated otherwise, the detectable signal-to-noise ratio evaluated ahead of an integrator will be smaller than when detection is performed by using single pulses.

There are many different methods of accomplishing integration. One method is the use of a feedback-loop delay line with a delay time equal to the interpulse period, so that signals (and noise) separated by exactly one pulse period will be directly added. Integration also occurs for visual detection by human radar operators if the phosphor of the cathode-ray-tube radar display (such as a PPI) has sufficient luminous persistence. In recent years, integration methods based on digital circuitry have become practical and are now perhaps the method of choice in many if not most cases.

The benefit of integration is a function of the number of pulses integrated. If integration is performed in the predetection stages of a receiver, ideally the addition of M equal-amplitude phase-coherent signal pulses will result in an output (integrated) pulse of voltage M times the single-pulse voltage. Adding M noise pulses, however, will result in an integrated noise pulse whose rms voltage is only $\sqrt{M}$ times as great as that of a single noise pulse if (as is true of ordinary receiver noise and many other types of noise) the added noise pulses are not phase-coherent. Therefore the signal-to-noise *voltage*-ratio improvement is, ideally, $M/\sqrt{M} = \sqrt{M}$. Consequently the signal-to-noise *power*-ratio improvement, and the reduction of the single-pulse minimum-detectable signal-to-noise power ratio, is equal to M.

Integration can also be performed after detection. In fact postdetection integration is used more commonly than is predetection integration, for reasons that will be explained, but the analysis of the resulting improvement is then much more complicated. After detection, the signals and the noise cannot be regarded as totally separate entities; the nonlinear process of detection produces an inseparable combination of signal and noise, so that one must then consider the comparison of signal-plus-noise to noise. As will be shown, the improvement that results from this type of integration is usually not as great as with ideal predetection integration of the same number of pulses. Nevertheless, postdetection integration produces worthwhile improvement. Moreover, "ideal" predetection integration is virtually unachievable because the echo fluctuation from most moving targets severely reduces the degree of phase coherence of successive received pulses. With rapidly fluctuating signals, in fact, postdetection integration will provide greater detectability improvement than does predetection integration, as discussed later in this section under the heading "Predetection Integration."

Number of Pulses Integrated. The number of pulses integrated is usually determined by the scanning speed of the antenna beam in conjunction with the antenna beamwidth in the plane of the scanning. The following equation can be used for calculating the number of pulses received between half-power-beamwidth points for an azimuth-scanning radar:

$$M = \frac{\phi\ \overline{\mathrm{PRF}}}{6\ \overline{\mathrm{RPM}} \cos \theta_e} \tag{2.16}$$

where ϕ is the azimuth beamwidth, $\overline{\text{PRF}}$ is the radar pulse repetition frequency in hertz, $\overline{\text{RPM}}$ is the azimuthal scan rate in revolutions per minute, and θ_e is the target elevation angle. This formula strictly applies only if $\phi/\cos\theta_e$, the "effective" azimuth beamwidth, is less than 360°. (At values of θ_e for which $\phi/\cos\theta_e$ is greater than 360°, the number of pulses computed from this formula will obviously be meaningless. Practically, it is suggested that it be applied only for elevation angles such that $\phi/\cos\theta_e$ is less than about 90°.) This formula is based on the properties of spherical geometry. The formula also assumes that the beam maximum is tilted upward at the angle θ_e, but it can be applied with negligible error if θ_e is only approximately equal to the beam tilt angle.

The formula for the number of pulses within the half-power beamwidth for an azimuth- and elevation-scanning radar (which can be applied with minor modification to a radar scanning simultaneously in any two orthogonal angular directions) is

$$M = \frac{\phi\theta\,\overline{\text{PRF}}}{6\omega_v t_v \overline{\text{RPM}}\cos\theta_e} \tag{2.17}$$

where ϕ and θ are the azimuth (horizontal-plane) and elevation (vertical-plane) beamwidths in *degrees*, θ_e is the target elevation angle, ω_v is the vertical scanning speed in degrees per second, and t_v is the vertical-scan period in seconds (including dead time if any). This formula should also be restricted to elevation angles for which $\phi/\cos\theta_e$ is less than about 90°. Here M is a function of the target elevation angle not only explicitly but also implicitly in that ω_v may be a function of θ_e.

Some modern radars, especially those capable of scanning by electronic means—i.e., without mechanical motion of the antenna—employ *step scanning*. In this method, the antenna beam is pointed in a fixed direction while a programmed number of pulses is radiated in that direction. Then the beam is shifted to a new direction, and the process is repeated. The number of pulses integrated in this scanning method is thus determined by the programming and not by the beamwidth. Also, the integrated pulses are then all of the same amplitude (except for the effect of target fluctuation), and so there is no *pattern loss* of the type described in Sec. 2.7. There is, however, a *statistical* loss if the target direction and the antenna beam maximum do not always coincide when the pulses are radiated.

Evaluation of Probabilities. As was mentioned in Sec. 2.2, if a threshold device is employed to make a decision as to the presence or absence of a signal in a background of noise, its performance can be described in terms of two probabilities: (1) the *probability of detection*, P_d, and (2) the *false-alarm probability*, P_{fa}. The threshold device is characterized by a value of receiver output voltage V_t (the *threshold*, equivalent to Marcum's[3] bias level), which, if exceeded, results in the decision report that a signal is present. If the threshold voltage is not exceeded at a particular instant, the detector reports "no signal."

There is always a definite probability that the threshold voltage will be exceeded when in fact no signal is present. The statistics of thermal random-noise voltage are such that there is a usually small but nonzero probability that it can attain a value at least equal to the saturation level of the receiver. (In the mathematical theory of thermal noise, there is a nonzero probability that it can attain any finite value, however large.) The probability that V_t is exceeded when no signal is present is the false-alarm probability. It is calculated from the equation

$$P_{fa} = \int_{V_t}^{\infty} p_n(v)dv \tag{2.18}$$

where $p_n(v)$ is the probability density function of the noise. The probability of detection is given by the same expression, with the probability density function that of the signal-noise combination (usually called *signal-plus-noise*, but the "addition" is not necessarily linear):

$$P_d = \int_{V_t}^{\infty} p_{sn}(v)dv \tag{2.19}$$

The signal-plus-noise probability density function $p_{sn}(v)$ depends on the signal-to-noise *ratio* as well as on the signal and noise statistics. Also, both p_n and p_{sn} are functions of the rectification law of the receiver detector and of any postdetection processing or circuit nonlinearities. Primarily, however, p_{sn} and therefore the probability of detection are functions of the signal-to-noise ratio. From Eq. (2.19), the variation of P_d with S/N can be determined. As would logically be assumed, it is a monotonic-increasing function of S/N for a given value of V_t. Similarly, the variation of P_{fa} as a function of V_t can be found from Eq. (2.18); it is a monotonic-decreasing function.

The method of applying these concepts to the prediction of radar range consists of four steps: (1) decide on a value of false-alarm probability that is acceptable (the typical procedure for making this decision will be described); (2) for this value of P_{fa}, find the required value of threshold voltage V_t, through Eq. (2.18); (3) decide on a desired value of P_d (in different circumstances, values ranging from below 0.5 up to as high as perhaps 0.99 may be selected); and (4) for this value of P_d and for the value of V_t found in step 2 find the required signal-to-noise ratio through Eq. (2.19). This requires evaluating the function $p_{sn}(v)$, taking into account the number of pulses integrated. Iteration is required, in this procedure, to find the value of D_0 corresponding to a specified probability of detection and number of pulses integrated. The value of D_0 thus found is the value to be used in the range equation [e.g., Eqs. (2.10) and (2.11)].

The process of finding the required value of D_0 for use in the range equation is greatly facilitated by curves that relate the number of pulses integrated to D_0 with P_{fa} and P_d as parameters. Many such curves have been published, and some representative ones are given as Figs. 2.3 through 2.7. The principal difficulty in computing them is determination of the probability density functions $p_n(v)$ and $p_{sn}(v)$ and in performing the requisite integrations. North[2] gives the exact functions that apply for single-pulse detection with a linear rectifier as detector and the approximations that apply when many pulses are integrated. The density functions appropriate to other situations, e.g., square-law detection and fluctuation of signals, are given by various authors.[3–10]

The decision as to the acceptable level of false-alarm probability is usually made in terms of a concept called *false-alarm time*, which will here be defined as the average time between false alarms. Other definitions are possible; Marcum[3] defines it as the time for which the probability of at least one false alarm is 0.5. However, the average time between false alarms seems a more practically useful concept. With it, for example, one can compute the average number of false alarms that will occur per hour, day, year, etc. With this definition, the false-alarm time is given by

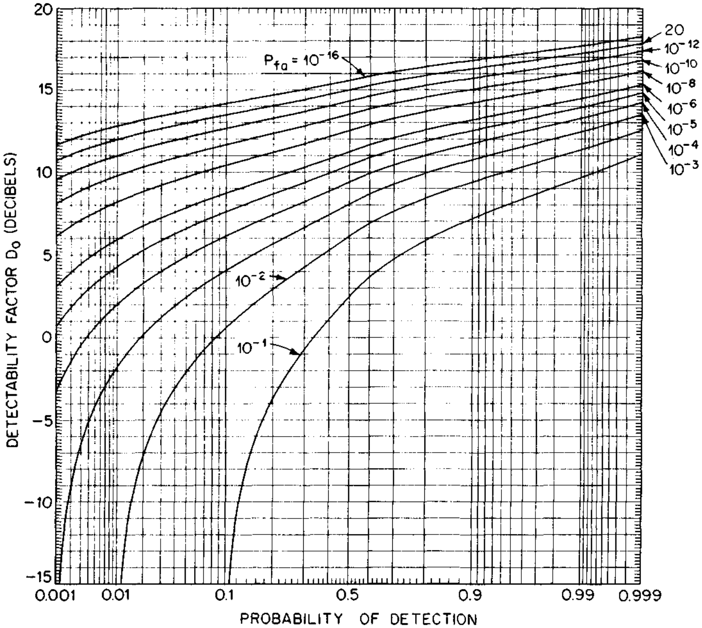

FIG. 2.3 **Required signal-to-noise ratio (detectability factor) for a single-pulse, linear-detector, nonfluctuating target as a function of probability of detection with false-alarm probability (P_{fa}) as a parameter.** *(From Ref. 13.)*

$$t_{fa} = \frac{M\tau}{P_{fa}} \tag{2.20}$$

where M is the number of pulses integrated and τ is the pulse duration.

This formula assumes that the integrator output is sampled at time intervals equal to τ. If range gates are employed and M pulses are integrated, if the ON time of the gate t_g is equal to or greater than the pulse length τ, and if there is some fraction of the time δ when no gates are open (*dead time*, e.g., just before, during, and after the occurrence of the transmitter pulse), then the formula is

$$t_{fa} = \frac{M\,t_g}{P_{fa}(1 - \delta)} \tag{2.21}$$

These false-alarm-time formulas assume that the receiver predetection noise bandwidth B_n is equal to or greater than the reciprocal of the pulse length and that the postdetection (video) bandwidth is equal to or greater than 0.5 B_n (as it usually is). These assumptions, usually met, amount to assuming that values of the noise voltage separated by the pulse duration are statistically independent; this independence occurs for times separated by $1/B_n$, sometimes called the Nyquist interval. Since ordinarily $B_n = 1/\tau$ and $t_g = \tau$, $1/B_n$ is sometimes used in place of τ or t_g in the false-alarm-time equations.

Marcum's false-alarm number n' is related to the false-alarm probability by the equation

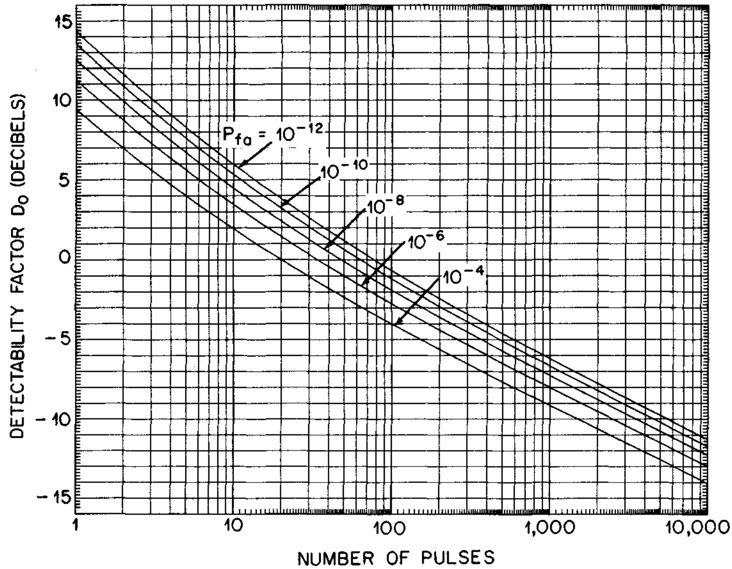

FIG. 2.4 Required signal-to-noise ratio (detectability factor) as a function of number of pulses noncoherently integrated, linear detector, nonfluctuating target, and 0.5 probability of detection. (*From Ref. 13.*)

$$1 - (1 - P_{fa})^{n'} = 0.5 \tag{2.22a}$$

For the usual large values of n' that are of interest, a highly accurate approximate solution of this equation for P_{fa} is

$$P_{fa} = \frac{\log_e 0.5}{n'} = \frac{0.6931}{n'} \tag{2.22b}$$

Fluctuating Target Cross Section. In general, the effect of fluctuation is to require higher signal-to-noise ratios for high probability of detection and lower values for low probability of detection than those required with nonfluctuating signals. Swerling has considered four cases, which differ in the assumed rate of fluctuation and the assumed statistical distribution of the cross section. The two assumed rates are (1) a relatively slow fluctuation, such that the values of σ for successive scans of the radar beam past the target are statistically independent but remain virtually constant from one pulse to the next, and (2) a relatively fast fluctuation, such that the values of σ are independent from pulse to pulse within one beamwidth of the scan (i.e., during the integration time).

The first of the two assumed distributions for the received-signal voltage is of the Rayleigh form,* which means that the target cross section σ has a probability density function given by

*The *Rayleigh* density function for a voltage v is

$$p(v) = \frac{2v}{r^2} e^{-v^2/r^2}$$

where r is the rms value of v.

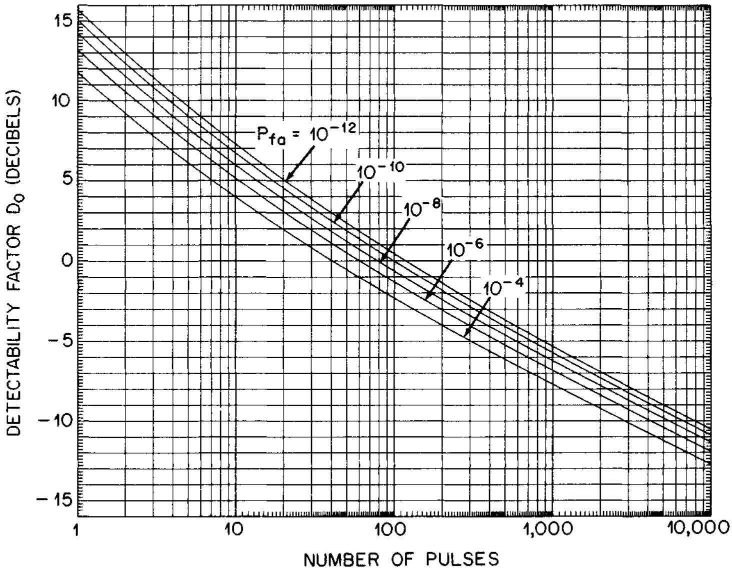

FIG. 2.5 Required signal-to-noise ratio (detectability factor) as a function of number of pulses noncoherently integrated, linear detector, nonfluctuating target, and 0.9 probability of detection. (*From Ref. 13.*)

$$p(\sigma) = \frac{1}{\bar{\sigma}} e^{-\sigma/\bar{\sigma}} \tag{2.23}$$

where $\bar{\sigma}$ is the average cross section. (This is a *negative-exponential* density function, but a target having this distribution is called a *Rayleigh target* because this distribution of σ produces a received signal *voltage* which is Rayleigh-distributed.) The second assumed cross-section density function is

$$p(\sigma) = \frac{4\sigma}{\bar{\sigma}^2} e^{-2\sigma/\bar{\sigma}} \tag{2.24}$$

The first distribution, Eq. (2.23), is observed when the target consists of many independent scattering elements of which no single one or few predominate. Many aircraft have approximately this characteristic at microwave frequencies, and large complicated targets are usually of this nature. (This result is predicted, for such targets, by the central limit theorem of probability theory.) The second distribution, Eq. (2.24), corresponds to that of a target having one main scattering element that predominates together with many smaller independent scattering elements. In summary, the cases considered by Swerling are as follows:

Case 1	Eq. (2.23), slow fluctuation
Case 2	Eq. (2.23), fast fluctuation
Case 3	Eq. (2.24), slow fluctuation
Case 4	Eq. (2.24), fast fluctuation

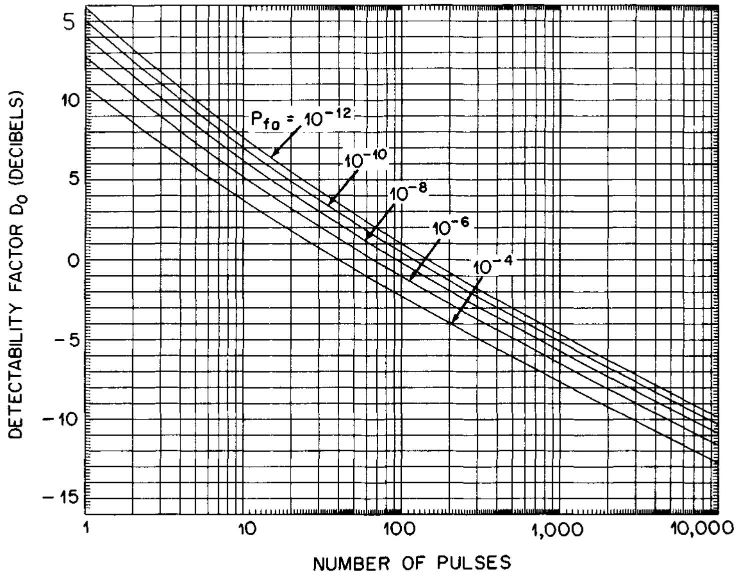

FIG. 2.6 Required signal-to-noise ratio (detectability factor) as a function of number of pulses noncoherently integrated, square-law detector, Swerling Case 1 fluctuating target, and 0.5 probability of detection. (*From Ref. 13.*)

The distribution of Eq. (2.24) is sometimes assumed for a small, rigid streamlined aircraft at the lower radar frequencies (e.g., below 1 GHz). Subsequent to Swerling's work, it has been found that many targets of the non-Rayleigh type are better represented by the so-called log-normal distribution, and analyses have been made for this case.[9]

Swerling's Case 1 is the one most often assumed when range prediction is to be made for a nonspecific fluctuating target. Results for this case are presented in Figs. 2.6 and 2.7. Curves for the other fluctuation cases and for additional values of detection probability are given in Refs. 13 and 14.

Detector Laws. A linear detector is a rectifier which has the rectification characteristic

$$\left.\begin{aligned} I_o &= \alpha V_i \qquad V_i \geq 0 \\ I_o &= 0 \qquad\;\; V_i < 0 \end{aligned}\right\} \tag{2.25}$$

where I_o is the instantaneous output current, V_i is the instantaneous input voltage, and α is a positive constant. Typical diodes approximate this law if V_i is larger than some very small value (e.g., a few millivolts). Such a diode is ordinarily used as the second detector of a superheterodyne radar receiver. Also, appreciable RF and IF gain usually precedes the second detector, so that the voltage applied to it is usually large enough (typically, an appreciable fraction of a volt) to ensure this "linear" type of operation.

A square-law detector is one that has the nonlinear characteristic

$$I_o = \alpha V_i^2 \tag{2.26}$$

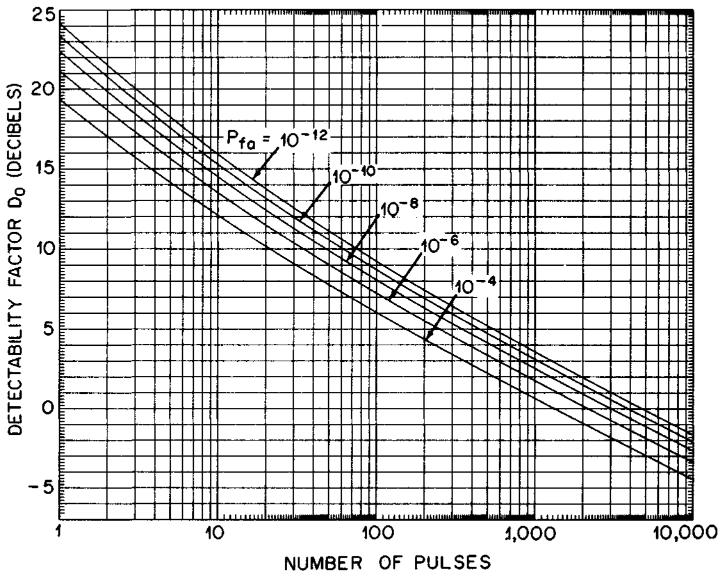

FIG. 2.7 Required signal-to-noise ratio (detectability factor) as a function of number of pulses noncoherently integrated, square-law detector, Swerling Case 1 fluctuating target, and 0.9 probability of detection. (*From Ref. 13.*)

Marcum[3] showed that a square-law detector is very slightly superior to a linear detector when many pulses are integrated, by about 0.2 dB. For a few pulses integrated, 10 or less, a linear detector is slightly superior—again, by about 0.2 dB or less. The mathematical analysis of probability of detection is somewhat more tractable when a square-law detector is assumed; this is probably its principal advantage.

The complexity of this matter is further compounded by the fact that, because of the statistics of the signal-noise superposition, in a *linear rectifier* there is a square-law relationship between the signal input voltage and the signal-plus-noise output voltage for small signal-to-noise ratios. This relationship becomes linear for large signal-to-noise ratios, as shown by Bennett,[22] North,[2] and Rice.[23] Because of this effect, it is sometimes erroneously thought that such a detector becomes square-law for small signal-to-noise *ratios*. But it is the input signal-plus-noise *voltage* V_i, and not the signal-to-noise ratio, that determines whether a diode rectifier is a linear or a square-law detector.

Curves for Visual Detection. The curves of Figs. 2.3 to 2.7 apply when the detection decision is based on an automatic threshold device as described. It is reasonable to suppose, however, that a human observer of a cathode-ray-tube display makes decisions in an analogous manner. That is, the equivalent of a threshold voltage (which would be a luminosity level for the PPI-scope type of display and a "pip-height" level for the A-scope display) exists somewhere in the observer's eye-brain system. This threshold, resulting in a particular false-alarm probability, is probably related to the observer's experience and personality: his or her innate cautiousness or daring. The probability of detection probably depends not only on the signal-to-noise ratio in relation to the threshold but on the observer's visual-mental acuity, alertness or fatigue, and experience. Consequently, curves calculated for an automatic threshold decision device cannot be assumed to apply accurately to the performance of a human observer of a cathode-ray-tube display. But such an assumption does not give grossly erroneous results, and it is justifiable when experimental human-observer data are not available or are of questionable accuracy.

Curves based on actual experiments with human observers, analogous to those of Figs. 2.4 through 2.7, are given in Ref. 14, Chap. 2, along with further discussion of visual detection.

Other Detection Methods. The discussion and results that have been presented have assumed perfect postdetection (video) integration of pulses prior to decision by an automatic threshold device. The noise statistics have been implicitly assumed to be those of ordinary receiver noise of quasi-uniform spectral density and (before detection) of gaussian probability density. A great many other detection procedures and signal-noise statistics are possible. Some of them are discussed in Ref. 14, Chap. 2.*

Predetection Integration. The results depicted in Figs. 2.3 through 2.7 apply for perfect *postdetection* (video) integration of a specified number of pulses. It was shown by North[2] that under ideal conditions *predetection* integration results in the smallest possible detectability factor and that for ideal predetection integration of M pulses the following relation holds:

$$D_0(M) = D_0(1)/M \tag{2.27}$$

That is, the minimum detectable signal-to-noise power ratio at the demodulator input terminals is improved, relative to single-pulse detection, by a factor exactly equal to the number of pulses integrated, M. For perfect postdetection integration, the improvement factor is generally less than M, asymptotically approaching $M^{1/2}$ as M becomes indefinitely large.

An exception occurs in the $M < 10$ region for fast-fluctuating targets and high probabilities of detection. For those circumstances the postdetection integration improvement factor can actually exceed M, and under the same circumstances predetection integration yields little or no improvement. The result of adding successive fast-fluctuating signals before detection, with virtually uncorrelated phases, is practically the same as that of adding noise voltages. Therefore there is virtually no integration improvement.

Predetection integration is also called *coherent integration*, because of its dependence on phase coherence of the integrated pulses, and postdetection integration is called *noncoherent integration*.

When integration is not perfect, as is always the case practically, if the value of D_0 used in the range equation is based on perfect integration, an imperfect-integration loss factor or factors must be included in the system loss factor L, as discussed in Sec. 2.7.

Although the full benefits of predetection integration are realizable only for nonfluctuating targets, some benefit can be achieved by predetection integration of a moderate number of *slowly* fluctuating targets. For such targets, the phase fluctuation from pulse to pulse is small. This type of integration is being employed to an increasing extent in modern systems when the utmost sensitivity is important and when fast fluctuation is not expected.

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Since radial target motion produces a frequency shift of the received-echo signal (doppler effect) which is proportional to the target's radial velocity, this shift must be taken into account if predetection integration is used. This is done in *pulse doppler radar* (Chap. 17).

Some radar systems that integrate many pulses utilize a combination of coherent and noncoherent integration when the phase stability of the received pulses is sufficient for some coherent integration but not great enough to allow coherent integration of the entire pulse train during the antenna on-target dwell time. If the total number of received pulses is N and M of them (with $M < N$) are coherently integrated and if the coherent integrator is followed by a noncoherent integrator, then (assuming an appropriate implementation and ideal integrations) the detectability factor will be

$$D_{0(M,N)} = D_0(N/M)/M \tag{2.28}$$

where $D_{0(M,N)}$ means the detectability factor for the assumed combination of coherent and noncoherent integration and $D_0(N/M)$ is the detectability factor for noncoherent integration of N/M pulses with no coherent integration (e.g., a value read from curves such as those of Figs. 2.4 through 2.7). As an example, if a train of $N = 24$ pulses is received and each set of $M = 8$ pulses is predetection- (coherently) integrated and if the predetection integrator is followed by a postdetection (noncoherent) integrator, the integration process produces at best a combined detectability-factor improvement corresponding to that of coherent integration of 8 pulses and noncoherent integration of 3 pulses.

2.5 SYSTEM NOISE TEMPERATURE

The concept of a *noise temperature* is derived from Nyquist's theorem,[24] which states that if a resistive circuit element is at temperature T (kelvins) there will be generated in it an open-circuit thermal-noise voltage given by

$$V_n = \sqrt{4kTRB} \quad \text{volts} \tag{2.29}$$

where k is Boltzmann's constant (1.38054×10^{-23} Ws/K), R is the resistance in ohms, and B is the bandwidth, in hertz, within which the voltage is measured (that is, the passband of an infinite-impedance voltmeter). The absence of the frequency in this expression implies that the noise is white—that the spectrum is uniform and extends to infinitely high frequency. But this also implies infinite energy, an obvious impossibility, indicating that Eq. (2.29) is an approximation. A more exact expression, which has frequency dependence, must be used if the ratio f/T exceeds about 10^8, where f is the frequency in hertz and T is the kelvin temperature of the resistor. Thus Eq. (2.29) is sufficiently accurate at a frequency of 30 GHz if the temperature is at least 300 K. The more accurate equations are given in Ref. 14 and in radio astronomy texts.

Available Power, Gain, and Loss. As thus defined, V_n is the open-circuit voltage at the resistor terminals. If an external impedance-matched load of resistance $R_L = R$ is connected, the noise power delivered to it will be

$$P_n = kTB_n \tag{2.30}$$

which does not depend on the value of R. This is of course also an approximation, but it is quite accurate at ordinary radar frequencies and temperatures. This matched-load power is called the *available* power.[17]

The concepts of *available power, available gain*, and its reciprocal, *available loss*, are assumed in all noise-temperature and noise-factor equations. These and other noise-temperature concepts are explained fully in Refs. 14, 17, and 25. Briefly, available power at an output port is that which would be delivered to a load that matches (in the complex-conjugate sense) the impedance of the source. Available gain of a two-port transducer or cascade of transducers is the ratio of the available power at the output port to that available from the source connected to the input port, with the stipulation that the available output power be measured with the actual input source (not necessarily impedance-matched) connected.

Noise Temperature. The usual noise that exists in a radar receiving system is partly of thermal origin and partly from other noise-generating processes. Most of these other processes produce noise which, within typical receiver bandwidths, has the same spectral and probabilistic nature as does thermal noise. Therefore it can all be lumped together and regarded as thermal noise. This is done, and the available-power level P_n is described by assigning to the noise a semifictitious "noise temperature" T_n, which is

$$T_n = P_n/(kB_n) \tag{2.31}$$

This is of course simply an inversion of Eq. (2.30), except that T in Eq. (2.30) refers to an actual (thermodynamic) temperature. The temperature defined by Eq. (2.31) is semifictitious because of the nonthermal origin of some of the noise. When this temperature represents the available-noise-power output of the entire receiving system, it is commonly called the system noise temperature or operating noise temperature,[17] and it is then used to calculate the system noise power and signal-to-noise ratio, as in Eqs. (2.4) to (2.6).

The Referral Concept. A receiving system can be represented as a cascade of two-port transducers, preceded by a source (the antenna) and terminated by a load. [However, in the discussion of system noise temperature, only those parts of the receiver that precede the detector (demodulator) are of significance, for the noise level at that point determines the signal-to-noise ratio for signal-detection-calculation purposes.]

Noise may arise at any and all points in this cascade, so that the noise level changes from point to point. The important quantity is the *output* noise power P_{no}. For purposes of signal-to-noise calculation, however, it is convenient to *refer* this output noise to the system input terminals. This is done by defining the *system noise temperature* T_s so that it satisfies the relation

$$kT_sB_n = P_{no}/G_0 \tag{2.32}$$

where G_0 is the overall-system available gain and B_n is the noise bandwidth of the system [Eq. (2.14)]. The output power P_{no} is thus "referred" to the system input (the antenna terminals), and T_s is actually the system *input* noise temperature. The product kT_sB_n is thus the system output noise power referred to the antenna terminals.

Each two-port transducer of the receiving-system cascade can be regarded as having its own effective input noise temperature T_e, representing its intrinsic available output noise power referred to its own input terminals. Here *intrinsic* means the power that the transducer would generate with a *noise-free* input termination of the same impedance as the actual input termination. Transducer output power is referred to the input terminals by dividing the output power by the available gain of the transducer.

For an N-transducer cascade, the system input noise temperature (with the antenna terminals considered to be the system input terminals) is then given by

$$T_s = T_a + \sum_{i=1}^{N} \frac{T_{e(i)}}{G_i} \tag{2.33}$$

Here T_a is the antenna noise temperature, representing the available noise power at the antenna terminals, and G_i is the available gain of the system between its input terminals and the input terminals of the ith cascaded component. (By this definition $G_1 = 1$ always.)

To illustrate these principles concretely, this formula will here be applied to a two-transducer cascade representing a typical receiving system (Fig. 2.8). The first transducer is the transmission line that connects the antenna to the receiver input terminals, and the second transducer is the predetection portion of the receiver itself. (As mentioned above, for purposes of signal-noise analysis subsequent portions of the receiver are not considered.) If desired, a many-transducer receiving system could be further broken down, with a preamplifier and possibly other units considered as separate elements of the cascade.

For this system, if the receiving-transmission-line noise temperature is represented by T_r and its loss factor is L_r $(=1/G_2)$ and if the receiver effective input noise temperature is T_e, Eq. (2.33) becomes

$$T_s = T_a + T_r + L_r T_e \tag{2.34}$$

It now remains to discuss evaluation of T_a, T_r, L_r, and T_e.

Antenna Noise Temperature. Antenna noise is the result of (1) noise in the form of electromagnetic waves received by the antenna from external radiating sources and (2) thermal noise generated in the ohmic components (resistive conductors and imperfect insulators) of the antenna structure. The product kT_aB_n is the noise power available at the antenna terminals within the receiver bandwidth.

This noise temperature is dependent in a somewhat complicated way on the noise temperatures of various radiating sources within the receiving-antenna pattern, including its sidelobes and backlobes. The concept of noise temperature of a radiating source is based on Planck's law or on the Rayleigh-Jeans approximation to it, analogous to the relationship of resistor noise temperature to Nyquist's theorem.

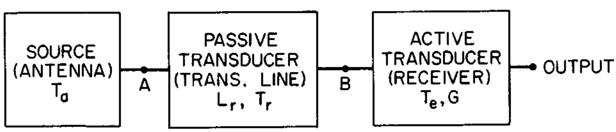

FIG. 2.8 Block diagram of a cascade receiving system.

The antenna noise temperature is not dependent on the antenna gain and beamwidth when a uniform-temperature source fills the beam. If the noise sources within the beam are of different temperatures, the resulting antenna temperature will be a solid-angle-weighted average of the source temperatures. The noise temperatures of most of the radiating sources that an antenna "sees" are frequency-dependent; therefore antenna temperature is a function of frequency. That is, antenna noise is not truly "white," but within any typical receiver passband it is virtually white.

In the microwave region, it is also a function of the antenna beam elevation angle, because in this region most of the "sky noise" is the result of atmospheric radiation. This radiation is related to atmospheric absorption, which is greater at low angles where the antenna beam sees a thicker slice of the lossy atmosphere than it does at higher angles.

Curves of antenna temperature for a lossless antenna are shown in Fig. 2.9, calculated for typical conditions.[14,25]

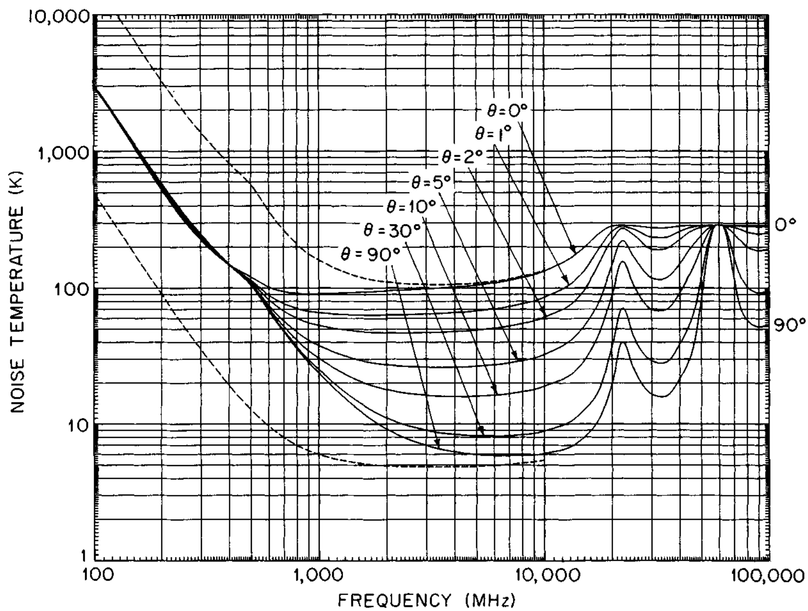

FIG. 2.9 Noise temperature of an idealized antenna (lossless, no earth-directed sidelobes) located at the earth's surface, as a function of frequency, for a number of beam elevation angles. Solid curves are for geometric-mean galactic temperature, sun noise 10 times quiet level, sun in unity-gain sidelobe, cool temperate-zone troposphere, 2.7 K cosmic blackbody radiation, and zero ground noise. The upper dashed curve is for maximum galactic noise (center of galaxy, narrow-beam antenna), sun noise 100 times quiet level, and zero elevation angle; other factors are the same as for the solid curves. The lower dashed curve is for minimum galactic noise, zero sun noise, 90° elevation angle. (The bump in the curves at about 500 MHz is due to the sun-noise characteristic. The curves for low elevation angles lie below those for high angles at frequencies below 400 MHz because of reduction of galactic noise by atmospheric absorption. The maxima at 22.2 and 60 GHz are due to water-vapor and oxygen absorption resonances; see Fig. 2.19.) *(From Ref. 13.)*

The curves of Fig. 2.9 apply to a lossless antenna that has no part of its pattern directed toward a warm earth. The lossless condition means that the curves represent only the noise received from external radiating sources. Therefore any thermal noise generated in the antenna must be added to the noise represented by these curves. In most practical cases, a ground noise-temperature component must also be added because part of the total antenna pattern is directed toward the ground. (This will be true because of sidelobes and backlobes even if the main beam is pointed upward.) But then also the sky-noise component given by Fig. 2.9 must be reduced somewhat because part of the total antenna pattern is not then directed at the sky. The reduction factor is $(1 - T_{ag}/T_{tg})$, where T_{ag} is the ground noise-temperature contribution to the total antenna temperature and T_{tg} is the effective noise temperature of the ground.

If α is the fraction of the solid-angle antenna power pattern subtended by the earth, then $T_{ag} = \alpha T_{tg}$. If the earth is perfectly absorptive (a thermodynamic blackbody), its effective noise temperature may be assumed to be approximately 290 K. A suggested conventional value for T_{ag} is 36 K, which would result if a 290 K earth were viewed over a π-steradian solid angle by sidelobes and backlobes averaging 0.5 gain (−3 dB). These sidelobes are typical of a "good" radar antenna but not one of the ultralow-noise variety.

Moreover, some practical antennas have appreciable ohmic loss, expressed by the loss factor L_a (Sec. 2.3). An additional thermal-noise contribution of amount $T_{ta}(1 - 1/L_a)$ then results, where T_{ta} is the thermal temperature of the lossy material of the antenna. However, the noise from external sources is then also reduced by the factor $1/L_a$. The total correction to the temperature values given by Fig. 2.9, to account for both ground-noise contribution and antenna loss, is then given by the following formula:

$$T_a = \frac{T_a'(1 - T_{ag}/T_{tg}) + T_{ag}}{L_a} + T_{ta}(1 - 1/L_a) \qquad (2.35a)$$

where T_a' is the temperature given by Fig. 2.9. For $T_{ag} = 36$ K and $T_{tg} = T_{ta} = 290$ K, this becomes

$$T_a = \frac{0.876\, T_a' - 254}{L_a} + 290 \qquad (2.35b)$$

and if $L_a = 1$ (lossless antenna), it further simplifies to

$$T_a = 0.876\, T_a' + 36 \qquad (2.35c)$$

Transmission-Line Noise Temperature. Dicke[26] has shown that if a passive transducer of noise bandwidth B_n connected in a cascade system is at a thermal temperature T_t and if its available loss factor is L, the thermal-noise power available at its *output* terminals is

$$P_{no} = kT_tB_n(1 - 1/L) \qquad (2.36)$$

A transmission line is a passive transducer. From Eq. (2.36) together with Eq. (2.31) and the definition of *input* temperature, it is deduced that the input noise temperature of a receiving transmission line of thermal temperature T_{tr} and loss factor L_r is

$$T_r = T_{tr}(L_r - 1) \tag{2.37}$$

(In this referral operation, multiplication by loss factor is equivalent to division by gain.) The receiving-transmission-line loss factor L_r is defined in terms of a CW signal received at the nominal radar frequency by the antenna. It is the ratio of the signal power available at the antenna terminals to that available at the receiver input terminals (points A and B, Fig. 2.8). A suggested conventional value for T_{tr} is 290 K.

Receiver Noise Temperature. The effective input noise temperature of the receiver T_e may sometimes be given directly by the manufacturer or the designer. In other cases, the *noise figure* F_n may be given. The relationship between the noise figure and the effective input noise temperature of the receiver or, in fact, of any transducer is given by[17]

$$T_e = T_0(F_n - 1) \tag{2.38}$$

where T_0 is, by convention, 290 K. In this formula F_n is a power ratio, not the decibel value that is usually given.

This formula is applicable to a *single-response* receiver (one for which a single RF input frequency corresponds to only one output or IF frequency and vice versa). Methods of computing noise temperatures when a double- or multiple-response receiver is used (e.g., for a superheterodyne receiver without preselection) are described in Refs. 17 and 25. Single-response receivers are ordinarily used in radar systems.

It is worth mentioning a point that has been well emphasized in the specialized literature of radio noise but is nevertheless easily overlooked. A receiver noise-temperature or noise-figure rating applies when a particular terminating impedance is connected at the receiver input. If this impedance changes, the noise temperature changes. Therefore, in principle, when a noise-temperature rating is quoted for a receiver, the source impedance should be specified, especially since the optimum (lowest) noise temperature does not necessarily occur when impedances are matched. However, when a receiver noise temperature is quoted without this impedance specification, it is presumable that the optimum source impedance is implied.

2.6 PATTERN PROPAGATION FACTOR

The pattern propagation factors F_t and F_r in the range equation account for the facts that (1) the target may not be in the beam maximum of the vertical-plane antenna pattern and (2) non-free-space wave propagation may occur. This single factor, rather than two separate factors, is designed to account for both of those effects. This is necessary because they become inextricably intertwined in the calculation of multipath interference, which is the most important non-free-space effect.

As will be seen, this effect can result in very considerable increase or decrease of the radar detection range compared with the free-space range. In this chapter, the basic ideas of pattern-propagation-factor calculation and some typical multipath-interference results will be presented. Additional details are given in Ref. 14, Chap. 6, and in Ref. 15.